Force response function on a moving sphere in a viscoelastic medium

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Contents

1 Abstract

The Intention of this thesis is to find an analytical solution or at least an approximation of the force response on a sphere moving in a viscoelastic medium. We use a two fluid model , c.f. [1], to describe a viscoelastic medium as the coupling of an elastic network to an incompressible Newtonian fluid. The coupling is done via a friction term, which is proportional to the relative motion between the elastic network and the fluid. Our study is limited to the regime of low Reynolds numbers which is justified for systems like biological tissues, polymere solutions or gels, where the moving object and the velocity are relatively small. A perturbation ansatz is used to decouple the differential equations of the two fluid model. Physically that means we consider only a small coupling of the Newtonian fluid and the elastic network. In zeroth order we are able to solve the stationary Navier Stokes equation identically to the derivation of the Stokes' law in [2], whereas the displacement field is solved by a radial symmetric field for a resting sphere in the origin. The time dependence of the field will then be added by substituting r with $r(t)$. The first order correction terms are calculated by the use of spherical harmonics and multipole expansion of the Green matrix of the equilibrium Navier Cauchy equation. We find that the resulting fields are not physically meaningful since they are divergent for large distances. In case of the first order correction term we find that this problem cannot be solved by adding homogeneous solutions of the differential equation due to properties of spherical harmonics. We argue that the divergent behaviour is due to a seeming incompatibility of the zeroth order velocity field to the zeroth order displacement field. This is caused by the long range of the r^{-1} decrease of the viscous velocity compared to the faster r^{-3} decrease of the velocity of the elastic network. Therefore we use the resulting fields only to get a rough estimation of the correction needed to the force response function. We discover that this correction seems physically plausible despite the underlying divergent fields. Surprisingly we do not find a R^2 proportionality in the correction terms of the force response function. In zeroth order we find the linear dependency on R and the first order correction is already cubic in R.

2 Physical concepts

At the beginning let us briefly remind ourselves of some of the physical concepts used to characterise viscoelastic systems. Since viscoelasticity is a combination of viscous and elastic behaviour we first want to consider each of it for itself. We start with the concept of viscosity.

2.1 Viscosity

In this subsection we follow the definitions of [2]. Viscosity characterises the internal friction of a fluid. Due to this internal friction, a viscous fluid will not return to its original shape after a deforming force is no longer applied to it. The deformation energy dissipates in the medium and can therefore no longer be used to return the viscous material to its original shape. The motion of a viscous fluid is described by the Navier Stokes Equation, which is given by

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right]=-\nabla P+\eta\Delta\mathbf{v}+\left(\zeta+\frac{1}{3}\eta\right)\nabla(\nabla\cdot\mathbf{v}) \tag{1}$$

where ρ is the density, P the pressure, η and ζ are coefficients of viscosity and $\mathbf{v}$ the velocity field. To describe a viscous system completely we also need the continuity equation

$$\frac{\partial\rho}{\partial t}=-\nabla\cdot\mathbf{v}. \tag{2}$$

The Reynolds number is a dimensionless quantity and is defined as

$$R=\frac{\rho w l}{\eta}, \tag{3}$$

where w is the main stream velocity and l the characteristic length. The latter has to be defined for the problem e.g the radius of the sphere. Physical problems are similar if the geometric shape of the problem and the Reynolds numbers are identical. The flow around a sphere with radius a is similar to the flow of another sphere with radius $a/2$ and a flow velocity of $2w$.
In the following discussion we consider the viscous medium to be incompressible. Therefore (2) becomes zero. Furthermore, we only consider the regime of small Reynolds numbers, which means that we can neglect the convection term $(\mathbf{v}\cdot\nabla)\mathbf{v}$. This simplifies (1) considerably to

$$\rho\dot{\mathbf{v}}=-\nabla p+\eta\Delta\mathbf{v}. \tag{4}$$

Before we come to elasticity, we will in the following subsection consider the derivation of the Stokes' Law.

2.2 The Stokes' Law

In the limit case of vanishing elasticity the solution for a viscoelastic system should reproduce the known Stokes' law. Therefore, we briefly discuss the force response of a purely viscous

incompressible fluid. The full calculation can be found in [2]. Here we will only look at it schematically.
We start with (4) and then assume a stationary stream i.e. $\dot{\mathbf{v}} = 0$. Therefore, we find

$$\Delta \mathbf{v} = -\nabla P, \tag{5}$$

$$\nabla \cdot \mathbf{v} = 0. \tag{6}$$

We write $\mathbf{v} = \mathbf{v}' + \mathbf{w}$, where $\mathbf{w}$ is the velocity of the fluid for $r \to \infty$. Because of (6) we can then write $\mathbf{v}' = \nabla \times \mathbf{A}$, where A has to be some axial vector and should be proportional to $\mathbf{r} \times \mathbf{u}$. With this we find that we can write $\mathbf{A} = f'(r)(\mathbf{n} \times \mathbf{w})$, where $\mathbf{n}$ is the unit vector in direction of $\mathbf{r}$ and $f'(r)$ is the derivative in respect to r of some scalar function $f(r)$.
Since $\mathbf{w}$ is per definition constant we can then write the velocity as

$$\mathbf{v} = \nabla \times \nabla \times (f\mathbf{w}) + \mathbf{w}. \tag{7}$$

Using $\nabla \times$ on both sides of (5) and on (7) we find that we just need to solve $\Delta^2 \nabla f = 0$ respectively

$$\Delta^2 f = const. = 0, \tag{8}$$

where the second equality follows from demanding that $\mathbf{v}'$ should vanish for large r and also all derivatives of it. The velocity is given by second order derivatives of f where (8) contains fourth order derivatives. The solution of (8) can be obtained by a separation ansatz followed by integration over r and considering the conditions for large r. We find that it is given by $f(r) = ar + \frac{b}{r}$. Plugging it into (7) and determining the constants a and b via the boundary condition that $\mathbf{v} = 0$ for $r = R$, we finally come to the solution for the velocity field

$$\mathbf{v} = -\frac{3R}{4}\frac{\mathbf{w} + \mathbf{n}(\mathbf{w}\cdot\mathbf{n})}{r} - \frac{R^3}{4}\frac{\mathbf{w} - 3\mathbf{n}(\mathbf{w}\cdot\mathbf{n})}{r^3} + \mathbf{w}, \tag{9}$$

where R is the radius of the sphere. The pressure can then be found to be $P = P_0 - \frac{3}{2}\eta R \frac{\mathbf{w}\cdot\mathbf{n}}{r^2}$.
The ith component of the force per unit area is given by

$$K_i = P n_i - \sigma'_{ik} n_k, \tag{10}$$

where σ'_{ik} are the components of the viscous stress tensor. The force acting from the fluid on the sphere can than be obtained by integrating over the surface of the sphere. Due to symmetry only the z-components contribute. Therefore, the force is

$$F = \oint \mathrm{d}f \mathbf{K} \cdot \mathbf{e_z} = \oint \mathrm{d}f(-P\cos\theta - \sigma'_{\theta r}\sin\theta), \tag{11}$$

where $\mathrm{d}f$ is the infinitesimal area element and $\mathbf{K}$ the Force per unit area. On the surface of the sphere the only two contributions are $\sigma'_{r\theta} = -\frac{3\eta}{2R} w \sin\theta$ and $P = -\frac{3\eta}{2R} w\cos\theta + P_0$ which was already considered at the second equality sign of (11). Therefore, we obtain

$$F = -\frac{3\eta w}{R} \oint R^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi = 6\pi R \eta w, \tag{12}$$

the right hand side of (12) is the Stokes' Law.

2.3 Elasticity

Elasticity is the ability of an body to return to its original shape after an external force stops acting on it. The deformation energy is stored in the body in contrary to a viscous material. The motion of an linear elastic body is described by the the so called Navier Cauchy Equation. It is given by

$$\mu\Delta\mathbf{u} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + \mathbf{F} = \rho\ddot{\mathbf{u}}, \tag{13}$$

where $\mathbf{F}$ is an external Force acting on the elastic material, ρ is the density of the body and λ and μ are called the Lamé constants, cf.[3]. The vectorfield $\mathbf{u}$ is called the displacement field, which describes the difference of the actual position of a particle to its equilibrium position.

2.4 Viscoelasticity

Materials that show elastic as well as viscous behaviour are called viscoelastic. Examples can be found in biological systems like cells but also in gels or polymere solutions.
As mentioned in an elastic body can the deformation energy is stored in the body, so when the external force is no longer acting on the body, it will return to its original form. Doing the same to a viscous fluid we will find that the fluid does not return to its original shape. The deformation energy dissipates in the fluid due to internal friction.
In a viscoelastic material both behaviours occur to some extend. This is in general time, temperature and frequency dependent, cf. [4].
Viscoelastic behaviour can be studied with rheological experiments, here an oscillating force is applied and one measures the response. The elastic component responds in phase with the applied shear, whereas the viscous component is out of phase. This can be described by introducing a complex shear modulus $G = G' + iG''$. The real part, often called storage modulus, corresponds to the elastic behaviour. The imaginary part, also called loss modulus, characterises viscous behaviour, cf. [5]. The so-called Generalized Einstein Stokes Relation (GSER) is given by [1]

$$\alpha(\omega) = \frac{1}{6\pi G(\omega) a} \tag{14}$$

where $G(\omega)$ is the complex shear modulus, a is the radius and $\alpha(\omega)$ is the compliance of the particle. The complex shear modulus $G(\omega)$ is defined as

$$G(\omega) = \mu - i\omega\eta. \tag{15}$$

We want to give some motivation to the GSER. This is not supposed to be a strict derivation but a simplified view on the subject. The result and idea behind it can be found in [1]. We try to motivate it in our own words. We start by imagining a particle, which is oscillating in a medium. Let the position be described by $x(t) = A\,\mathrm{e}^{-i\omega t}$. Its velocity is then $\dot{x}(t) = -i\omega A\,\mathrm{e}^{-i\omega t} = -i\omega x(t)$. If we consider a pure viscous fluid we will need to apply an oscillating force $F(\omega, t)$ to compensate the energy loss due to the friction. The forces acting on the particle are then

$$m\ddot{x}(t) = -6\pi\eta v(t)R + F(w, t), \tag{16}$$

where we inserted the Stokes' law to describe the force acting on the particle at a given time t. If we neglect the inertial effects, i.e. we set $m\ddot{x}$ to be zero and plug in the expression for the velocity we can show that the position can be described as

$$x(t) = \frac{F(w,t)}{6\pi R(-i\omega\eta)} = \frac{F(\omega,t)}{6\pi RG(\omega)}, \tag{17}$$

were we have written the $-i\omega\eta$ as the complex shear modulus. We could now consider the ratio $x(t)/F(\omega,t)$, which characterises how the particle responds to the applied force. The imaginary character of the found compliance means that there is a phase lag in the response. This is a characteristic property of viscous fluids. The idea of the GSER is now to generalize this to viscoelastic media. The response of an elastic medium is almost instantaneous i.e. there will be no phase lag between the applied force and the reaction of the particle. Therefore we expect the ratio to be real. Since viscoelastic media have elastic as well as viscous properties we expect that the shear modulus is a complex number, where the imaginary part is the phase lagging viscous component and the real part corresponds to the elastic behaviour. For rotating particles it can be shown that the compliance fulfills the GSER excepts for inertial effects, which can be no longer ignored for high frequencies cf. [6]. There exist different methods to model viscoelasticity. In our case we will use the two fluid model in a continuum limit, which will be discussed in the next subsection.

2.5 The two fluid model

For the description of our viscoelastic medium we use a two fluid model as defined in [1],[7]. Here the viscoelastic medium is modelled via the coupling of an elastic network to an incompressible viscous fluid. The elastic network is described by the Navier-Cauchy-Equation, see (13). We consider the elastic network macroscopically as an isotropic and homogeneous system, so we can describe it in continuum limit. This is valid for length scales larger than the mesh size ξ of the network. The incompressible and viscous fluid is described by Navier Stokes Equation, see (4). Both equations in the two fluid model are coupled via a friction term $\Gamma(\dot{\mathbf{u}} - \mathbf{v})$, where $\dot{\mathbf{u}}$ corresponds to the velocity of a particle of the elastic network. The model is completed by the demand that the solution as whole is incompressible. The resulting set of partial differential equations is given by

$$\rho\ddot{\mathbf{u}} - \mu\Delta\mathbf{u} - (\lambda+\mu)\,\nabla(\nabla\cdot\mathbf{u}) = -\Gamma\,(\dot{\mathbf{u}} + \mathbf{v}) + \mathbf{f}^{\mathbf{u}}, \tag{18}$$

$$\rho_F\dot{\mathbf{v}} - \eta\Delta\mathbf{v} + \nabla P = \Gamma\,(\dot{\mathbf{u}} - \mathbf{v}) + \mathbf{f}^{\mathbf{v}}, \tag{19}$$

$$\nabla\cdot[(1-\Phi)\mathbf{v} + \Phi\dot{\mathbf{u}}] = 0, \tag{20}$$

where λ and μ are the Lamé coefficients, Γ the coupling coefficient, ρ the density of the elastic network , ρ_F the density of the viscous fluid, P the pressure, $\mathbf{f}^{\mathbf{u}}$ and $\mathbf{f}^{\mathbf{v}}$ are external forces acting on the network or the fluid and ϕ is the ratio of the volumes $\frac{V_{network}}{V_{fluid}}$. The coupling constant Γ is proportional to the ratio $\frac{\eta}{\xi^2}$, where ξ is the mesh size of the elastic medium. The coupling can be understand as follows: The elastic network can move relative to the background fluid. The relative motion results in friction between the two media. the friction force is proportional to the relative velocity between the two media. For small velocities the coupling is only weak and the system behaves like two separate media with only little perturbations. For larger velocities the coupling becomes rather strong and the system can be considered to be a single incompressible viscoelastic medium [6]. With that in mind the inverse proportionality of the friction constant to the squared mesh size makes sense, since a finer meshed network means more windage. Some applications of the two fluid model can be found in microrheolgy, where it can be used for modelling e.g rotating particle rheology [6] or for investigating translatorial microrheology [1]. In the rotating case the solution are analytically exact, which motivates why we to use the two fluid model in our own problem.

3 The sphere in a viscoelastic medium

After the discussion of the physcial background concepts we now turn our attention to the actual goal of this thesis. The goal is to find out if we can find an analytical solution or at least an approximation for the force response on a sphere moving stationary through a model viscoelastic medium, which we describe with a two fluid model. Similar to the derivation of the Stokes' Law [2] in a purely viscous and incompressible fluid, we consider the sphere fixed in the origin and let the viscoelastic medium flow around it in direction of the z-axis. In general we can calculate the force acting on the sphere by integrating the force per unit area $\mathbf{K}$ over the surface of the sphere, so

$$\mathbf{F} = \oint \mathrm{df}\mathbf{K}. \tag{21}$$

Since we employ the two fluid model, we can separate the force acting on a unit area of the sphere in a viscous and elastic part, i.e [3],[2]

$$K_i = K_i^u + K_i^v = -\sigma_{ik}^v n_k - \sigma_{ij}^u n_j, \tag{22}$$

where σ^v is the viscous stress tensor, σ^u is the elastic stress tensor and $\mathbf{n}$ is the normal vector of the unit area. Here we need to regard that the normal vector points in the opposite direction of $\mathbf{e_r}$. The components of the stress tensor can be found in [3] and [2]. Due to the symmetry of the sphere only the projection of the z-axis does not vanish. We can therefore write

$$F = \oint \mathrm{df}(-\sigma_{ik}^v n_k - \sigma_{ij}^u n_j)\mathbf{e_i} \cdot \mathbf{e_z}. \tag{23}$$

The symmetry suggests the use of spherical coordinates. We remember that the normal vector $\mathbf{n}$ points in the opposite direction of $\mathbf{e}_r$. This means that the only non-zero component of $\mathbf{n}$ is $n_r = -1$. Therefore we need to determine the elements σ_{rr} and $\sigma_{r\theta}$ of the viscous and elastic stress tensors. We find for the force

$$F = \oint \mathrm{df}(\sigma_{rr}^v \cos\theta - \sigma_{r\theta}^v \sin\theta + \sigma_{rr}^u \cos\theta - \sigma_{r\theta}^u \sin\theta), \tag{24}$$

where we used $\mathbf{e_r} \cdot \mathbf{e_z} = \cos\theta$ and $\mathbf{e}_\theta \cdot \mathbf{e_z} = -\sin\theta$. The components of the viscous stress tensor are determined by the velocity field and its derivatives, whereas the elements of the elastic stress tensor are defined via the displacement field and its derivatives. To conclude our calculation of the force response we have to first find the velocity field and displacement field for our problem. Those fields must fulfill the partial differential equations of the two fluid model (18), (19) and (20). We follow a similar approach as in [7] and consider Φ much smaller than one. The external forces $\mathbf{f^u}$ and $\mathbf{f^v}$ are set to zero. Furthermore, since we assume the velocity to be stationary $\dot{\mathbf{v}} = 0$. The resulting set of partial differential equations as a consequence is

$$\mu\Delta\mathbf{u} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) = \Gamma(\dot{\mathbf{u}} - \mathbf{v}), \tag{25}$$

$$\eta\Delta\mathbf{v} - \nabla P = -\Gamma(\dot{\mathbf{u}} - \mathbf{v}), \tag{26}$$

$$\nabla \cdot \mathbf{v} = 0. \tag{27}$$

Solving (25), (26) and (27) might still be pretty sophisticated. Therefore, we will try an approach by a perturbation ansatz.

3.1 Perturbation ansatz

First we write (25) and (26) by introducing an artificially small perturbation coefficient ϵ as

$$\mu \Delta u - (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) = \epsilon\Gamma(\dot{\mathbf{u}} - \mathbf{v}), \tag{28}$$

$$\eta \Delta \mathbf{v} = -\nabla P - \epsilon\Gamma(\dot{\mathbf{u}} - \mathbf{v}), \tag{29}$$

This might be physically interpreted as a weak coupling between elastic network and viscous fluid. We then write $\mathbf{u}$ and $\mathbf{v}$ as perturbation series

$$\mathbf{u} = \mathbf{u_0} + \sum_{i=1}^{\infty} \epsilon^i \mathbf{u_i}, \tag{30}$$

$$\mathbf{v} = \mathbf{v_0} + \sum_{i=1}^{\infty} \epsilon^i \mathbf{v_i}. \tag{31}$$

If we now plug (30) in (28) and similarly (31) in (29) and sort in order of ϵ we get in 0^{th} order

$$\mu \Delta \mathbf{u_0} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u_0}) = 0, \tag{32}$$

$$\eta \Delta \mathbf{v_0} = -\nabla P, \tag{33}$$

i.e. two completely decoupled equations. The physical meaning is that in zeroth order the two media behave like complete separated from each other. The solution for the velocity in this case is already known and is given by (19). For higher order we then find a recursive formula for the displacement field $\mathbf{u}$ and velocity field $\mathbf{v}$

$$\mu \Delta \mathbf{u_i} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u_i}) = \Gamma(\dot{\mathbf{u}}_{\mathbf{i-1}} - \mathbf{v_{i-1}}) \tag{34}$$

$$\eta \Delta \mathbf{v_i} = -\Gamma(\dot{\mathbf{u}}_{\mathbf{i-1}} - \mathbf{v_{i-1}}), \tag{35}$$

where the next order is coupled to the previous solutions.In principle, we should be able to solve this recursively provided that we first find a solution for $\mathbf{u_0}$.

3.2 Zeroth order approximation of the displacment field

We assume that the displacment field looks in 0^{th} order similar to the displacment field of the elastic network caused by a resting sphere in the origin. Therefore, we first solve the Navier Cauchy Equation for this case. In this stationary scenario the second time derivative of $\mathbf{u}$ is zero and there is no external force acting on the elastic network. As a consequence, (13) becomes

$$\mu\Delta\mathbf{u} + (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) = 0. \tag{36}$$

Due to the spherical symmetry of the problem we anticipate that $\mathbf{u}(\mathbf{r})$ is just a function of the distance $r = |\mathbf{r}|$. Therefore we can write $\mathbf{u}$ as $\mathbf{u}(r) = -\nabla g(r)$, where $g(r)$ is a scalar function of r. If we plug this ansatz in (36) we obtain

$$\mu\Delta(\nabla g(r)) + (\lambda + \mu)\nabla(\nabla \cdot \nabla g(r)) = \nabla(\lambda + 2\mu)\Delta g(r) = 0. \tag{37}$$

From here it follows that $(\lambda + 2\mu)\Delta g(r) = const.$. The Laplacian operator for a spherical symmetrical scalar function is given by $\frac{2}{r}\partial_r + \partial_r\partial_r$. We demand that $\mathbf{u}$ should vanish for large r. Therefore the constant must be set to zero, because the derivative of $g(r)$ in respect to r is $|\mathbf{u}|$. The second derivative is then the change of $|\mathbf{u}|$ as a function of the distance. Since for large r this quantity should also vanish it becomes clear that we must set the constant to zero. This leads us to a Laplacian equation for $g(r)$ with the general solution

$$g(r) = \frac{C}{r} + b, \tag{38}$$

where we can set b to zero, since it vanishes anyway after differentiating. Plugging the solution in our Ansatz for $\mathbf{u}(r)$ we find

$$\mathbf{u}(r) = -\nabla g(r) = \frac{C}{r^2}\mathbf{e_r} \tag{39}$$

The constant C is determined by boundary conditions. Usually the displacement vector contains the information where a particle of the medium is displaced to when it is at point $\mathbf{r}$. We use here a different interpretation and declare that point $\mathbf{r}$ is the point where the particle is due to displacement and $\mathbf{u}(\mathbf{r})$ tells us how far it was displaced from its original point. In other words the position $\mathbf{r}$ is the endpoint of the displacement vector and not the foot. In this case we set the boundary condition so that on the surface of the sphere the displacement must be proportional to the radius of the sphere. Therefore $C = cR^3$, where c is a material specific proportionality factor.

We solved the Navier Cauchy Equation for a resting sphere, which serves as our 0^th order approximation for $\mathbf{u}$. For the iteration to first and higher orders however we need the time derivative of $\mathbf{u}$. When the sphere is moving through the medium, the distance r to a particle of the medium changes over time. Similarly if we consider the scenario from the point of view of the sphere a particle of the network will increase its distance over time. Therefore, we replace r with $r(t)$ in (39). The sphere is travelling along the negative z-axis or respectively the medium is streaming upwards the z-axis and the sphere is resting. We therefore write for the z component $z(t) = wt$ where w is the stationary velocity. Obviously, since there are no time derivatives in (36) the expression

$$\mathbf{u}(r(t)) = \frac{C}{r(t)^2}\mathbf{e_r}, \tag{40}$$

is still a solution of the partial differential equation. When taking the time derivative of the equation above, we need to remember that also the reference points of the elastic medium travel with $w\mathbf{e_z}$. We need to add this to the formal time derivative and find then

$$\dot{\mathbf{u}}_\mathbf{0} = \frac{Cw}{r^3}\mathbf{e_z} - \frac{3Cwz}{r^5}\mathbf{r} + w\mathbf{e_z}. \tag{41}$$

From here we can start to determine the first order corrections term.

3.3 Formal solution of first order approximation

For the first order correction we need to solve the following set of partial differential equations. One way to go is to find respective Green functions and then convolve them with the friction terms. In this section we will give the expressions of the formal solution and will later try to get an explicit solution.

$$\mu\Delta\mathbf{u_1} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u_1}) = \Gamma(\dot{\mathbf{u}}_\mathbf{0} - \mathbf{v_0}), \tag{42}$$

$$\eta\Delta\mathbf{v_1} = -\Gamma(\dot{\mathbf{u}}_\mathbf{0} - \mathbf{v_0}). \tag{43}$$

First we will focus on (43), which resembles a Poisson equation. To construct the solution of each component, we need to find the Greens function which fulfills our boundary conditions. We write the Green function as

$$g(\mathbf{r}-\mathbf{r}') = g_h(\mathbf{r}-\mathbf{r}') + h(\mathbf{x},\mathbf{x}'), \tag{44}$$

where $g_h(\mathbf{r}) = \frac{-1}{4\pi|\mathbf{r}|}$ is the known Green function of the Laplacian Equation. $h(\mathbf{r})$ is a support function which fulfills the Laplacian Equation outside of the sphere and on the surface becomes equal to $-g_h(\mathbf{r})$. Assuming such support function exists we can then write the solution as the convolution of the Green function with the friction term as

$$v_i^1 = \frac{1}{\eta}\int g(\mathbf{r}-\mathbf{r}')(\dot{u}_i^0 - v_i^0)(\mathbf{r}')\mathrm{dV}', \tag{45}$$

In a similar way we can solve (42). According to [8] the Green matrix for the equilibrium Navier Cauchy Equation is given by the Kelvin Somigliani Matrix

$$G_{ij}^h(\mathbf{x}) = \frac{1}{8\pi\mu(\lambda+2\mu)}\left((\lambda+3\mu)\frac{\delta_{ij}}{|\mathbf{x}|} + (\lambda+\mu)\frac{x_i x_j}{|\mathbf{x}|^3}\right), \tag{46}$$

where δ_{ij} ist the Kronecker Delta. To prove that (46) is indeed the Green's matrix of (42) , we have to show that

$$A_{klim}G_{ij} = \delta_{jk}\delta(\mathbf{x}), \tag{47}$$

where $\delta(\mathbf{x})$ is the Dirac Delta Distribution and $A_{klim} = \lambda\delta_{kl}\delta_{im} + \mu(\delta_{ki}\delta_{lm} + \delta_{km}\delta_{li})$ is the differential operator of (42) expressed in components. For $|\mathbf{x}| \neq 0$ it can be shown that $A_{klim}G_{ij} = 0$ by direct calculation, cf.[8]. However, at the origin, we have to be more careful with the calculation. Using A_{klim} on G_{ij} gives several terms with differentials whose behaviour at the origin is not safe to say. Nevertheless, we can integrate each term over a sphere with radius r in the limit $\lim_{r\to 0}$. Then we can use in each case the Gauss Theorem to switch from volume integrals to surface integrals. As a result one can show that G_{ij} indeed is the wanted Green function. As explained before we will need a support function $\mathbf{h}(\mathbf{r})$ in order to fulfill boundary condition. Assuming such function exists we write $G_{ij}(\mathbf{r}) = G^h_{ij}(\mathbf{r}) + h_{ij}(\mathbf{r})$. We then may write our solutions for (42) and (43) as

$$\mathbf{u_1} = \int \mathbf{G}(\mathbf{r}-\mathbf{r}')\Gamma\dot{\mathbf{u}}_\mathbf{0} - \mathbf{v_0}(\mathbf{r}')\mathrm{dV}' \tag{48}$$

$$v^1_i = \frac{\Gamma}{\eta}\int g(\mathbf{r}-\mathbf{r}')(\dot{u}^0_i - v^0_i)(\mathbf{r})\mathrm{dV}', \tag{49}$$

For an explicit solution of v_i we take a different approach, which will now be discussed in the following section.

3.4 Determination of first order velocity field

We now want to develop the solution for v_1 using spherical harmonics. We will discuss the calculation for v^1_x here, the other two components work similar and we will give the explicit expressions for them later. We want to use the fact that every solution of the Laplacian Equation can be expanded in spherical harmonics to construct a homogeneous solution. The latter will be used such that the boundary condition holds i.e. $v^1_x = 0$ on the surface of the sphere. First, we will need to find a special solution for v^1_x. The x component of $\mathbf{v}$ is given by

$$\Delta v^1_x = -\left(\frac{A}{r} + \frac{B}{r^3}\right)\cos\theta\sin\theta\cos\phi = -h(r)f(\theta,\phi) \tag{50}$$

where $A = \frac{\Gamma}{\eta}\frac{Rw}{r}$ and $B = \frac{\Gamma}{\eta}(\frac{3}{4}R^3w - 3Cw)$. We can write $f(\theta,\phi)$ as a linear combination of the two spherical harmonics Y_{21} and $\mathrm{Y}_{2,-1}$. If we assume that v^1_x is square integrable we are able to write it as a spherical harmonics expansion

$$\Delta v^1_x = \Delta\left(\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_{lm}\mathrm{Y_{lm}}f_l(r)\right) = -h(r)(a_{21}\mathrm{Y}_{21} + \mathrm{a}_{2,-1}\mathrm{Y}_{2,-1}), \tag{51}$$

where c_{lm} is the expansion coefficient and $f_l(r)$ is some radial function. The Laplacian operator in spherical coordinates is given by $\Delta = (\frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}) + \frac{1}{r^2}\Delta_{\theta,\phi}$. We can use the linearity of the Laplacian to use it on each summand and find

$$\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_{lm}\mathrm{Y_{lm}}\left(\Delta_r f_l(r) - \frac{l(l+1)f_l(r)}{r^2}\right) = -h(r)(a_{21}\mathrm{Y}_{21} + \mathrm{a}_{2,-1}\mathrm{Y}_{2,-1}), \tag{52}$$

where we used the property of spherical harmonics that $\Delta_{\theta,\phi}\mathrm{Y_{lm}} = l(l+1)\mathrm{Y_{lm}}$. Considering the right hand side of the equation above, we recognize that the only two summands in the expansion that can contribute are those given on the right hand side. Since the $f_l(r)$ term on the left hand side is independent of m we can factor it out, and divide both sides by the linear combination of the spherical harmonics on the right hand side. This leaves us with

$$\Delta_r f(r) - \frac{6f(r)}{r^2} = h(r) = -\frac{A}{r} - \frac{B}{r^3}. \tag{53}$$

Since the righ thand side is a polynom in r we assume that $f(r)$ is a polynom in r as well. Both the Laplacian and the $\frac{1}{r^2}$ term reduce the degree of the polynomial by 2. With an educated guess we find that the solution should look like

$$f(r) = ar + \frac{b}{r}. \tag{54}$$

Plugging this in (53) we find that this indeed solves the equation if we set $a = \frac{A}{4}$ and $b = \frac{B}{6}$. Therefore the special solution for v_x^1 can be written as

$$v_{x,s}^1 = \left(\frac{A}{4}r + \frac{B}{r}\right) \sin\theta \cos\theta \cos\phi. \tag{55}$$

The problem with (55) is that it diverges for large r. Solving that issue would be possible if we found a homogeneous solution of the Laplacian with the same angular dependence and an asymptotic behavoiur like $\frac{A}{4}r$. As we will see soon this seems not possible due to the characteristics of the spherical harmonics.
Any solution of the Laplacian equation can be written in the form

$$f(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(a_{lm} r^l + \frac{b_{lm}}{r^{l+1}} \right) \mathrm{Y_{lm}}. \tag{56}$$

It can be shown by applying the Laplacian operator that indeed every summand cancels out. However since the two possible exponents of r are given for every l, we cannot construct a solution with the same angular properties as our special solution and the same approach to infinity. Therefore we cannot fix the divergent behaviour of our solution by adding homogeneous solutions. The physical interpretation of the solution for v_x^1 would be that the velocity goes to infinity for large r, which is not physically reasonable. Furthermore any further calculation of higher order correction terms of v would also diverge. The Problem originates from the $1/r$ term of the velocity field $\mathbf{v}_0$. The latter was solved for a purely viscous fluid. A suggestion would be that the long range behaviour is no longer viable when the elastic network is present. We can estimate that v_0 must decrease at least with r^{-3} so that we no longer have this divergent behaviour. This suggests that one would need to search for a different solution for the 0^{th} order viscous velocity field.

3.5 First order viscous force response correction

Although the velocity field is not a valid physical solution for large distances, we want to investigate which force corrections we would get in that case. We can add a homogeneous solution which fulfills the boundary condition on the sphere's surface. There are two possible choices for the homogeneous function. We can pick the r^l or the r^{-l-1} term. At this point none of these stand out in particular. The decreasing solution would be the top choice if our special solution were not divergent. The r^l solution is increasing too fast and would over-compensate the special solution. We will compute both solutions and compare the results. Our general solution is

$$v_1^x = \frac{\Gamma R^2 w}{16\eta}\left[3\frac{r}{R} - (2+8c)\frac{R}{r} - \alpha_{\pm 1}(r)(1-8c)\right]\cos\theta\sin\theta\cos\phi, \tag{57}$$

where $\alpha_{+1}(r) = \frac{r^2}{R^2}$ corresponds to the r^l solution and $\alpha_{-1}(r) = \frac{R^3}{r^3}$ corresponds to the r^{-l-1} solution. The other components of $\mathbf{v_1}$ can be calculated in a similar way. We find

$$v_1^y = \frac{\Gamma R^2 w}{16\eta}\left[3\frac{r}{R} - (2+8c)\frac{R}{r} - \alpha_{\pm 1}(r)(1-8c)\right]\cos\theta\sin\theta\sin\phi, \tag{58}$$

$$\begin{aligned} v_1^z = \frac{\Gamma R^2 w}{16\eta}\Bigg(&\left[3\frac{r}{R} - (2+8c)\frac{R}{r} - \alpha_{\pm 1}(r)(1-8c)\right]\cos\theta^2 + \frac{7r}{R} + \\ &+ \frac{1}{3}((2+8c))\left(\frac{R}{r} - \frac{\alpha_{\pm 1}(r)}{R}\right) + \alpha_{\pm 1}(r) - 8\beta_{\pm 1(r)}\Bigg), \end{aligned} \tag{59}$$

where $\beta_{+1}(r) = 1$ corresponds to the r^l solution and $\beta_{-1}(r) = \frac{R}{r}$ belongs to the r^{-l-1} solution. We are now able to calculate the first order correction for the viscous force component. For the calculation of the force we need to integrate over the sphere's surface. In this case spherical coordinates are more pleasant to use. Transforming (57),(58) and(59) into spherical coordinates gives

$$v_r^1 = \frac{\Gamma R^2 w}{16\eta}\left[10\frac{r}{R} - 8\beta_{\pm 1}(r) - 2\alpha_{\pm 1}(r) - \frac{16}{3}\left(c + \frac{1}{4}\right)\left(\frac{R}{r} - \alpha_{\pm 1}(r)\right)\right]\cos\theta \tag{60}$$

$$v_\theta^1 = -\frac{\Gamma R^2 w}{16\eta}\left[7\frac{r}{R} - 8\beta_{\pm 1}(r) + 1\alpha_{\pm 1}(r) + \frac{16}{6}\left(c + \frac{1}{4}\right)\left(\frac{R}{r} - \alpha_{\pm 1}(r)\right)\right]\sin\theta \tag{61}$$

At this point we can compute the needed viscous stress tensor component $\sigma_{rr}^v = 2\eta\partial_r v_r^1$ and $\sigma_{r\theta}^v = \eta(1/r\partial_\theta v_r^1 + \partial_r v_\theta^1 + v_\theta^1/r)$.For the definition of the viscous stress tensor components cf. [2]. Since we calculate them at the surface of the sphere v_r and v_θ^1 are zero due to the boundary condition. Only the derivatives do not vanish. It follows that in our case the tensor component $\sigma_\theta^v = \eta\partial_\theta v_\theta^1$. We plug them into (24) and solve the integral, of course at this point we just calculate the viscous contribution i.e. the elastic stress tensor σ^u is considered zero. After some rearrangement we find the first correction of the viscous force contribution to the force response

$$F_1^v = 6\pi\eta w R \begin{cases} \frac{\Gamma R^2}{\eta}\left(\frac{17}{36} + \frac{4c}{9}\right) & \text{for the } r^l \text{ solution,} \\ \frac{\Gamma R^2}{\eta}\left(\frac{26}{27} - \frac{4c}{27}\right) & \text{for the } r^{-l-1} \text{ solution.} \end{cases} \tag{62}$$

The proportionality of the force in (62) to the respective parameters w, R and Γ is in both case similar. The prefactors factor differ though. We can use the result as an estimation on how the first order response depends on the parameters w, R and Γ, because this dependency seems physically plausible. As expected the force scales linearly with the velocity. The friction constant itself is inversely proportional to the squared mesh size ξ^2, which means that the force depends on the mesh size in the same way. This is reasonable, because the viscous fluid experiences less drag while moving relative to the network. This leads in return to a reduction of the force needed to move the sphere with a certain velocity w. What comes as a surprise is that we do not find a R^2 proportionality in the correction term but already find the R^3 proportionality in the first order correction. However, for further investigations we need to consider the elastic force contribution to the correction as well. Therefore, we postpone further discussion of the result and first dedicate ourselves to the first order displacement field.

3.6 Elastic force response correction by multipole expansion

We want to expand the Greens Matrix in a Taylor series in order to get at least some feeling of how the displacement field contributes. Due to symmetry, the monopole does not contribute to the force response and can therefore be neglected. The multidimensional Taylor expansion of $G_{ij}(\mathbf{r}-\mathbf{r}')$ around $\mathbf{r}=0$ can be written as

$$G_{ij}(\mathbf{r}-\mathbf{r}') = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}(\mathbf{r}\cdot\nabla)^n G_{ij}(\mathbf{r}-\mathbf{r}')\Big|_{\mathbf{r}=0}, \tag{63}$$

where the $|_{\mathbf{r}=0}$ means that the derivative of G_{ij} is evaluated at the position $\mathbf{r}=0$. Using this Taylor series expansion we can now calculate $\mathbf{u_1}$ in a sense of multipole moments.

$$\mathbf{u_i^1}(r) = \sum_{n=0}^{\infty} \int \mathrm{dV}' \frac{(-1)^n}{n!}(\mathbf{r}\cdot\nabla)^n G_{ij}(\mathbf{r}-\mathbf{r}')\Big|_{\mathbf{r=0}} \cdot \Gamma(\dot{u}_j^0 - v_j^0)(\mathbf{r}') \tag{64}$$

We will investigate the first three multipole moments for the force response. We already argued that the monopole will not contribute to the force response and start therefore directly with the dipole moment of the displacement field. We find

$$u_{i,(d)}^1 = -\int_R^\infty \mathrm{dr}' \int_0^\pi \mathrm{d}\theta' \sin\theta \int_0^{2\pi} \mathrm{d}\phi' r'^2 x_l x_k G_{ij,kl}(\mathbf{r}')\Gamma_j(\mathbf{r}') \tag{65}$$

where we introduced $\Gamma_j = \Gamma(\dot{u}_0 - v_0)_j$. The $1/r$ term in G_{ij} gives after taking the derivative in respect to x_k a term of form $\frac{-x_k}{r'^3}$. We transform the x_k' respectively in spherical coordinates. Which means we get for $x' = r\cos\phi\sin\theta$, $y' = r\sin\theta\sin\phi$ and $z' = r\cos\theta$. For better overview we write the components of Γ_j explicitly as

$$\Gamma_x = \left(\frac{3Rw}{4}\frac{1}{r} - \frac{3R^3w - 12Cw}{4r^3}\right)\cos\theta\sin\theta\cos\phi \tag{66}$$

$$\Gamma_y = \left(\frac{3Rw}{4}\frac{1}{r} - \frac{3R^3w - 12Cw}{4r^3}\right)\cos\theta\sin\theta\sin\phi \tag{67}$$

$$\Gamma_z = \left(-\frac{Rw}{r} + \frac{4Cw + R^3w}{4r^3}\right)(1 - 3\cos\theta^2) + \left(\frac{Rw}{r}\right) \tag{68}$$

The angular integrals can be interpreted as scalar products of spherical harmonics. Since the x_k' correspond to the spherical harmonics $\mathrm{Y}_{1,\pm1}$ or $\mathrm{Y}_{1,0}$ and the Γ_i correspond to $Y_{2,m}$ we can see that we do not get a single contribution from the first term in G_{ij}. The derivative of the second summand in G_{ij} is proportional to $\frac{\delta_{jk}x_i+\delta_{jk}x_j}{r'^3} - 3\frac{x_i x_j x_k}{r'^5}$. We can use the same argument as before for the first two summands. The third term is a little bit more complex. Direct calculation shows however that the terms will also vanish after the angular integration. Retrospectively we probably could have argued that we will not get a dipole contribution since the Γ_j do not have dipole moments. We move on to the quadupole moments and find

$$u^1_{i,(q)} = \int_R^\infty \mathrm{d}r' \int_0^\pi \mathrm{d}\theta' \sin\theta \int_0^{2\pi} \mathrm{d}\phi' r'^2 x_k \partial_k x_l \partial_l G_{ij}(\mathbf{r}')\Gamma_j(\mathbf{r}'), \tag{69}$$

where we used the convention that the comma in the index denotes a derivative in respect to the indices after comma. We calculate the derivatives of G_{ij}, which gives us three different forms of terms : $1/r$, $\frac{x'_a x'_b}{r'5}$ and $\frac{x'_a x'_b x'_c x'_d}{r'7}$, where the indices were chosen just as an example. The $1/r$ like terms are only relevant for Γ_z, since it is orthogonal to Γ_x and Γ_y. The second term only contributes if it corresponds to the respective spherical harmonic of the Γ_i after transforming the x'_a again into spherical coordinates. Which means e.g. for Γ_x only the terms of form $\frac{x'z'}{r'5}$ do not vanish under the integral. And similar then for the other two. The last term is not so obvious. But in case of uncertainty we can always calculate the respect angular integral and see if it vanishes or not. However we can save a lot of time if we first consider the integral of ϕ over 2π and then test only the remaining integrals of θ over π. We will not discuss the whole calculation here but only consider the case of $u^1_{x,(q)}$. the other two are done likewise. In this case we only find three contributing terms.

$$u^1_{x,(q)} = xz \int dV' \left(G_{xx,xz}\Gamma_x + G_{xy,yz}\Gamma_y + G_{xz,xz}\Gamma_z\right)(\mathbf{r}'). \tag{70}$$

The explicit calculation does not give us more insight which is why we will just write down the solutions of $u^1_{x,(q)}$ and the other two right in the following. We find

$$u^1_{x,(q)} = \Gamma w \left(\frac{1}{2} - c\right)\left(\frac{12\pi}{15}A - \frac{4\pi}{15}B\right) r^2 \cos\theta \sin\theta \cos\phi, \tag{71}$$

$$u^1_{y,(q)} = \Gamma w \left(\frac{1}{2} - c\right)\left(\frac{12\pi}{15}A - \frac{4\pi}{15}B\right) r^2 \cos\theta \sin\theta \sin\phi, \tag{72}$$

$$u^1_{z,(q)} = \Gamma w \left(\frac{1}{2} - c\right)\left[\left(\frac{12\pi}{15}A - \frac{4\pi}{15}B\right)\cos\theta^2 - \left(\frac{4\pi}{15}A + \frac{28\pi}{35}B\right)\right] r^2, \tag{73}$$

where $A = \frac{\lambda+3\mu}{8\pi\mu(\lambda+2\mu)}$ and $B = \frac{\lambda+\mu}{8\pi\mu(\lambda+2\mu)}$. Those are the special solutions for the quadrupole moments of the first order correction of the displacement field. Actually we still need to find a homogeneous solution in order to have the general solution for given boundary conditions. However, at this point we just want to estimate how the correction in the force response would look like and calculate the force response for the special solution. We recognize again that our special solution is not finite for large r. It might be possible to fix this behaviour with a respective homogeneous solution but as we said we will now try to get just a rough estimate of the force response correction.
We transform the quadrupole displacement field $\mathbf{u}^{\mathbf{1}}_{(\mathbf{q})}$ into spherical coordinates and find

$$u^1_{r,(q)} = \left(\frac{1}{2} - c\right)\left(\frac{4\pi}{15}A + \frac{28\pi}{35}B\right) r^2 w \sin\theta, \tag{74}$$

$$u^1_{\theta,(q)} = \left(\frac{1}{2} - c\right)\left(\frac{8\pi}{15}A + \frac{16\pi}{15}B\right) r^2 w \cos\theta. \tag{75}$$

With those we can now compute the elastic stress tensor components σ^u_{rr} and $\sigma^u_{r\theta}$. The components are defined as

$$\sigma^u_{rr} = (\lambda + 2\mu)\partial_r u_r + \lambda\left(\frac{1}{r}\partial_\theta u_\theta + \frac{1}{r\sin\theta}\partial_\phi, u_\phi + \frac{1}{\tan\theta}\frac{u_\theta}{r} + \frac{2u_r}{r}\right) \tag{76}$$

$$\sigma^u_{r\theta} = \mu\left(\frac{1}{r}\partial_\theta u_r + \partial_r u_\theta - \frac{u_\theta}{r}\right), \tag{77}$$

cf. [3]. Since we have no ϕ dependence the derivative in respect to ϕ vanishes. The force is then calculated via (24). Plugging in our displacement field and computing the integral we find after some rearrangement

$$F^1_{u,(q)} = 4\pi w R^3 \Gamma \frac{\lambda(2\lambda + \mu)}{45\mu(\lambda + 2\mu)}\left(\frac{1}{2} - c\right). \tag{78}$$

The sign as well as the prefactor might change if a homogeneous solution is added. However, we can assume that the general solution should at least have the same dependence from R, Γ and w. Therefore, we conclude that like the first correction term of the viscous force contribution, our elastic force correction is linear proportional to the flow velocity w and the cube of the radius of the sphere R^3. The latter means that the first correction scales with the volume of the sphere. ¡it is surprising that the first order correction already scales withe volume rather than the surface of the sphere. The dependence of the parameters w , R^3 and Γ seems physical plausible. One indeed expects that the force correction should be larger in case of a faster moving sphere and also if the sphere is bigger. The latter is because the sphere would have to displace more of the elastic network when moving through the medium. The friction constant Γ is inversely proportional to the squared mesh size of the elastic network. A larger mesh size therefore means a smaller resistance, which is also plausible. For any deeper understanding we would first need to find the general solution for precisely defined boundary conditions for the correction of the displacement field. But even if we do so there is still the issue with the divergent viscous velocity field, which might indicate a general incompatibility of our two initial zeroth order fields. Therefore, it might be more gainful to analyse if there might be other, better suited starting fields. As mentioned the long range of the $1/r$ term in the viscous velocity field might be origin of our compatibility issue. So finding a shorter ranged solution of the Navier Stokes Equation could give the opportunity to investigate the force response deeper. In retrospective we could also use the Green matrix of the Navier Cauchy Equation to compute the zeroth order displacement field, convolution with an assumed force distribution comes to mind. The work on a short range viscous velocity field seems however more gainful.

4 Summary

At this point we want to sum up what we did, and give some prospects. Our goal was to study the force response function of a sphere which moves uniformly in a viscoelastic medium. We started with the two fluid model, where a viscous, incompressible Newtonian fluid is coupled to an elastic network. The coupling is due to friction which is caused by relative motion of fluid and network. We limited our study to the regime of low Reynolds numbers which is justified for systems like biological cells, gels or polymere solutions where the object and the velocity is relatively small and considered similar to the derivation of the Stokes' law the problem as stationary. We approached the system of partial differential equations with a perturbation ansatz which corresponds to a small coupling of viscous fluid and elastic network. One additonal Therefore, we were able to decouple the differential equations of the velocity field and displacement field in the zeroth order. Higher orders are then only coupled to the previous velocity and displacement field. One additional benefit is that the condition that in the limit of vanishing elasticity the force response should reproduce the Stokes' law are automatically fulfilled with our ansatz.

For the displacement field, we showed that a radial symmetric field for a fixed sphere in the origin solves the Navier Cauchy Equation. The velocity of a particle of the network was then calculated by assuming the z-component to be time dependent and taking then the time derivative of our displacement field. For the zeroth order viscous velocity field we used the solution of the velocity field for a purely viscous medium which flows around a resting sphere.

We gave the expression of the formal solution of the first order correction terms of both the displacement field and the velocity field. In general, one can use the Green matrix for the displacement field and convolve it with our friction term. For the velocity field one can use the Green function of the Laplace operator. In both cases a support function can be employed in order to fulfill the boundary conditions.

However, we approached the actual task of solving the equation differently. In case of the velocity field we wrote each component in terms of a function of distance multiplied by an angular function. We discovered that each component could be represented exactly by a finite number of spherical harmonics. The properties of spherical harmonics were used to find a special solution for each component. However, we recognized that the special solution was not quite physical meaningful due to its divergent behaviour. This could not be solved by adding homogeneous solutions to the special solution, because the spherical harmonics fix the exponent of the radial function. We figured out that the problem was the relative long range of the $1/r$ term of the viscous velocity field and assumed that in presence of the elastic network the field would need to decrease faster. We decided to study how the force response on the sphere would look like anyway, maybe just to get a rough estimation of the dependence of the parameters. We found that although the velocity field itself was not quite plausible the force response makes physically sense.

After that, we wanted to see how the elastic network contributes to the first order force correction. The differential equation was solved by a multipole expansion series of the Green matrix which was then convoluted with the friction term. From symmetry follows that we do not get a contribution from the monopole moment. We did not find a contributing dipole moment though, either. The first contribution was in the order of quadrupoles and although the discovered solution has again divergent behaviour. The quadrupole moment of the displacement field was used to compute an estimation of the force response correction. We employed only the special solution to get a rough estimate and found that the contribution of the first order force response correction is scaling similar like the viscous force response correction what means both force correction are proportional to the volume of the sphere, the velocity and inversely proportional to the mesh size. This is physically reasonable, although the used correction of the velocity field and displacement field are not. Therefore, it is questionable if we can draw any real conclusion from our calculated force response. What came as a surprise though is that we did not find any force correction terms proportional to R^2, meaning scaling with the surface of the sphere. In the case of the elastic network the R^2 proportionality would appear in the multipole expansion in the order of dipoles. Since the friction term has no dipole moment, the integrals over ϕ and θ did vanish. Maybe one could try to get some sort of dipole moment if a relaxation time for the elastic network were introduced so that the network does not immediately relax after passing the sphere. For more detailed conclusions one would ave to study this problem further.

At this point we want to give some ideas for improvement. Since we discovered a seemingly incompatibility of our zeroth order velocity field and displacement we argue that for a deeper analysis one should try to find a faster decreasing velocity field. The velocity field should at least fall like r^{-3}. However, it is uncertain if such a velocity field exists as a solution for the stationary Navier Stokes Equation. One could also try to modify the initial displacement field although we think it is more gainful to focus on fixing the velocity field. Another idea would be to approach the coupled system of the two fluid model from a different angle instead the perturbation ansatz.

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